CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Secondary Education Certificate®

SYLLABUS

ADDITIONAL MATHEMATICS

CXC 37/G/SYLL 18

Effective for examinations from May–June 2021

CSEC®

Correspondence related to the syllabus should be addressed to:

The Pro-Registrar
Caribbean Examinations Council
Caenwood Centre
37 Arnold Road, Kingston 5, Jamaica

Telephone Number: + 1 (876) 630-5200
Facsimile Number: + 1 (876) 967-4972
E-mail Address: cxcwzo@cxc.org
Website: www.cxc.org

Contents

Additional Mathematics Syllabus

♦ RATIONALE

The Caribbean society is an integral part of an ever-changing world. The impact of globalization on most societies encourages this diverse Caribbean region to revisit the education and career opportunities of our current and future citizens. A common denominator of the Caribbean societies is to create among its citizens a plethora of quality leadership with the acumen required to make meaningful projections and innovations for further development. Further, learning appropriate problem-solving techniques, inherent to the study of mathematics, is vital for such leaders. Mathematics promotes intellectual development, is utilitarian and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. Therefore, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes which are precursors for this dynamic world.

This Additional Mathematics course provides a variety of topics with related attributes which will enable Caribbean students to reason logically using the prior knowledge gained from the **CSEC®** Mathematics. Candidates are expected to enter this course of study with a solid foundation of algebraic knowledge and mathematical reasoning.

This course of study incorporates the features of the Science, Technology, Engineering, and Mathematics (STEM) principles. On completion of this syllabus, students will be able to make a smooth transition to higher levels of study in Mathematics and other related subject areas, or move on to career choices where a deeper knowledge of the general concepts of Mathematics is required. This Additional Mathematics course includes fundamentals of Pure and Applied Mathematics. Through the use of learner centred teaching and assessment approaches, this course of study will enable students to develop and enhance twenty-first century skills including *critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research, information communication and technological skills which are integral to everyday life and for life-long learning. Students will be* exposed to the underlying concepts of Mathematics to foster a deeper understanding and greater appreciation of the subject. This course thus provides insight into the exciting world of advanced mathematics, thereby equipping students with the tools necessary to approach any mathematical situation with confidence.

This syllabus will contribute to the development of the Ideal Caribbean Person as articulated by the CARICOM Heads of Government in the following areas: "demonstrate multiple literacies, independent and critical thinking and innovative application of science and technology to problem solving." Such a person should also "demonstrate a positive work attitude and value and display creative imagination and entrepreneurship." In keeping with the UNESCO Pillars of Learning, on completion of this course the study, students will learn to do, learn to be and learn to transform themselves and society.

♦ AIMS

The syllabus aims to:

1. build upon those foundational concepts, techniques and skills acquired at the **CSEC®** Level and form linkages to areas of study at the Advanced Proficiency Level;

2. enhance ways of learning Mathematics;

3. stimulate further curiosity and analytical thinking in deriving solutions to problems which may not necessarily be solved by a single/unique approach;

4. develop abilities to reason logically;

5. *develop skills such as, critical and creative thinking, problem solving, logical reasoning, modelling, collaboration, decision making, research, and information communication and technological skills which are integral to everyday life and for life-long learning;*

6. develop positive intrinsic mathematical values, such as, accuracy and rigour;

7. connect Mathematics with other disciplines such as Science, Business and the Arts; and,

8. *integrate Information and Communications Technology (ICT) tools and skills in the teaching and learning processes.*

♦ PREREQUISITES OF THE SYLLABUS

Any person with a good grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (**CSEC®**) course in Mathematics, or equivalent, should be able to undertake this course. However, successful participation in this course will also depend critically on the possession of good verbal and written communication skills.

♦ ORGANISATION OF THE SYLLABUS

The syllabus is arranged as a set of topics, and each topic is defined by its specific objectives and content. It is expected that students would be able to master the specific objectives and related content after successfully pursuing a course in Mathematics during five years of secondary education.

The topics are arranged in four sections as follows:

Section 1 - Algebra, Sequences and Series

Section 2 - Coordinate Geometry, Vectors, and Trigonometry

Section 3 - Introductory Calculus

Section 4 - *Probability and Statistics*

♦ SUGGESTIONS FOR TEACHING THE SYLLABUS

For students who complete **CSEC®** Mathematics in the fourth form year, Additional Mathematics can be done in the fifth form year. Alternatively, students may begin Additional Mathematics in fourth form *(Grade 10)* and sit both **CSEC®** Mathematics and Additional Mathematics examinations at the end of form five *(Grade 11).* Students may even do the **CSEC®** Additional Mathematics as an extra subject simultaneously with **CAPE®** *Pure Mathematics and or Applied Mathematics* Unit 1 in the Sixth Form *(Grades 12 and 13).*

♦ CERTIFICATION

The syllabus will be examined for certification at the General Proficiency Level.

In addition to the overall grade, there will be a profile report on the candidate's performance under the following headings:

1. Conceptual Knowledge (CK);

2. Algorithmic Knowledge (AK); and,

3. Reasoning (R).

♦ DEFINITION OF PROFILES

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

Conceptual knowledge	-	the ability to **recall**, **select** and **use** appropriate facts, concepts and principles in a variety of contexts.
Algorithmic knowledge	-	the ability to **manipulate** mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.
Reasoning	-	the ability to **select**, **use** and **evaluate** mathematical models and **interpret** the results of a mathematical solution in terms of a given real-world problem, and to **engage** problem-solving.

♦ FORMAT OF THE EXAMINATIONS

Paper 01
(1 hour 30 minutes)

This Paper will consist of 45 multiple-choice items. Each question is worth one mark. The 45 marks will be weighted to 60 marks. The items will be distributed as presented in the table below:

Section	*Topics*	*No. of items*	*Total*
1	*Algebraic Operations*	*2*	*15*
	Quadratics	*4*	
	Inequalities	*2*	
	Surds, Indices and Logarithms	*4*	
	Series	*3*	
2	*Co-ordinate Geometry*	*3*	*15*
	Vectors	*3*	
	Trigonometry	*9*	
3	*Differentiation*	*5*	*10*
	Integration	*5*	
4	*Probability & Statistics*	*5*	*5*
	Total		***45***

Paper 02
(2 hours 40 minutes)

This Paper will consist of six compulsory questions as follows:

***Two** questions from Section I, Algebra, Sequences and Series, each worth 15 marks; **One** question from Section II, Coordinate Geometry, Vectors and Trigonometry worth 20 marks; **Two** questions from Section III, Introductory Calculus, each worth 15 marks; and **One** question from Section IV, Probability & Statistics worth 20 marks. The marks allocated to the sections are shown below.*

Sections		*No. of questions*	*Marks*			*Total*
			CK	*AK*	*R*	
1	*Algebra, Sequences and Series*	*2*	*4*	*16*	*10*	*30*
2	*Coordinate Geometry, Vectors and Trigonometry*	*1*	*4*	*10*	*6*	*20*
3	*Introductory Calculus*	*2*	*4*	*18*	*8*	*30*
4	*Probability & Statistics*	*1*	*4*	*10*	*6*	*20*
Total Marks			*16*	*54*	*30*	*100*

SCHOOL-BASED ASSESSMENT (SBA) – Paper 031

This paper comprises a project requiring candidates to demonstrate the practical application of Mathematics in everyday life. In essence, it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools. The topic(s) chosen may be from any section or a combination of different sections of the syllabus.

The project may require candidates to collect data, or may be theory-based, requiring solution or proof of a chosen problem. Group SBAs are encouraged, with optimal group size of six– each student must actively participate in completing the tasks.

Candidates who have registered to write both **CSEC®** Additional Mathematics and **CSEC®** Mathematics in the same sitting may opt to transfer marks from the **CSEC®** Additional Mathematics SBA to **CSEC®** Mathematics. Candidates are not allowed to transfer marks from **CSEC®** Mathematics to **CSEC®** Additional Mathematics.

The total marks for Paper 031 is 30 (weighted to 40 marks) and will contribute 20 per cent toward the final assessment. See Guidelines for School-Based Assessment on pages 33–56.

Paper 032 (Alternative to Paper 031), examined externally.

This paper is an alternative to Paper 031 and is intended for private candidates. This paper comprises one question. The given topic(s) may be from any section or combination of different sections of the syllabus. The duration of the paper is 1 ½ hours.

WEIGHTING OF PAPER AND PROFILES

The percentage weighting of the examination components and profiles is as follows:

Table 1 - Percentage Weighting of Papers and Profiles

PROFILES	***PAPER 01***	***PAPER 02***	***PAPER 03***	***TOTAL %***
Conceptual (CK)	9 (12)	16	09 (12)	40 (20%)
Algorithmic Knowledge (AK)	24 (30)	54	12 (16)	100 (50%)
Reasoning (R)	12 (18)	30	09 (12)	60 (30%)
TOTAL [%]	45 (60) 30%	100 50%	30 (40) 20%	200 100%

♦ REGULATIONS FOR RESIT CANDIDATES

*For **CSEC®** candidates, SBA scores can be carried forward only ONCE and only during the year immediately following the first sitting. In order to assist candidates in making decisions about whether or not to reuse a moderated SBA score, the Council will continue to indicate on the preliminary results if a candidate's moderated SBA score is less than 50 per cent in a particular subject. Candidates reusing SBA scores should register as "Resit candidates" and must provide the previous candidate number when registering. These candidates must rewrite Papers 01 and 02 of the examination for the year in which they re-register.*

Resit candidates may enter through schools, recognized educational institutions or the Local Registrar's Office.

♦ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 032.

Private candidates must be entered through institutions recognized by the Council.

♦ MISCELLANEOUS SYMBOLS

$=$	is equal to
$\neq$	is not equal to
$<$	is less than
$\leq$	is less than or equal to (is not greater than)
$>$	is greater than
$\geq$	is greater than or equal to (is not less than)
$\equiv$	is identical to
$\approx$	is approximately equal to
$\propto$	is proportional to
∞	infinity
$\in$	*is a member of*
$\notin$	*is not a member of*
$\mathbb{N}$	*the set of Natural Numbers*
R	*the set of Real Numbers*
R^+	*the set of positive Real Numbers*

Operations

$\sum_{i=1}^{n} x_i$	$x_1 + x_2 + x_3 + \ldots + x_n$

Calculus

$\frac{dy}{dx}$, y'	*the first derivative of y with respect to x*
$\frac{d^n y}{dx^n}$, y^n	*the n^{th} derivative of y with respect to x*
$f'(x)$, $f''(x)$, $\cdots$, $f^{(n)}(x)$	*the first, second, …, n^{th} derivatives of $f(x)$ with respect to x*

$\dot{x}, \ddot{x}$	the first and second derivatives of x with respect to time
lgx	the logarithm of x to base 10
$\int y\ dx$	the indefinite integral of y with respect to x
$\int_a^b y\ dx$	$\int_u^h y$ the definite integral of y with respect to x between the limits $x = a$ and $x = b$

Probability and Statistics

$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
S	the possibility sample space
$P(A)$	the probability of the event A occurring
$P(A')$	the probability of the event A not occurring
$P(A\|B)$	the conditional probability of the event A occurring given the event B has occurred

Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{\mathbf{AB}}$	the vector represented in magnitude and direction by the directed line segment AB
$\left\|\overrightarrow{\mathbf{AB}}\right\|$,	the magnitude of $\overrightarrow{\mathbf{AB}}$
$\hat{\mathbf{a}}$	a unit vector in the direction of **a**
$\|\mathbf{a}\|$	the magnitude of **a**
$\mathbf{a}.\mathbf{b}$	the scalar (dot) product of **a** and **b**
i, j	unit vectors in the direction of the Cartesian coordinate axes, *x* and *y* respectively

$\begin{pmatrix} x \\ y \end{pmatrix}$ $x\mathbf{i} + y\mathbf{j}$

Kinematics

x	displacement
$v, \dot{x}$	velocity
$a, \dot{v}, \ddot{x}$	acceleration

♦ LIST OF FORMULAE

As prerequisite knowledge, students would be expected to know formulae from the ***CSEC®*** *Mathematics Syllabus as may be applicable.*

Arithmetic Series

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Series

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$ $\quad S_\infty,$ $\quad -1 < r < 1$ or $|r| < 1$

Circle: $x^2 + y^2 + 2fx + 2gy + c = 0$ $\quad (x + f)^2 + (y + g)^2 = r^2$

Vectors

$\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|}$ $\quad \cos\theta = \frac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|}$ $\quad |\mathbf{v}| = \sqrt{(x^2 + y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$

Trigonometry

$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$

Calculus

$$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$$

$$\frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\cos ax = -a\sin ax$$

$$s = \int v\,dt$$

$$v = \int a\,dt$$

Kinematics

$$v = \frac{dx}{dt} = \dot{x};$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$$

Statistics

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\bar{x})^2$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

♦ USE OF ELECTRONIC CALCULATORS

Candidates are expected to have an electronic non-programmable calculator and are encouraged to use such a calculator in Paper 02. Candidates will also be allowed to use a calculator in Papers 01 and 032.

Guidelines for the use of electronic calculators are listed below.

1. Silent, electronic hand-held calculators may be used.

2. Calculators should be battery or solar powered.

3. Candidates are responsible for ensuring that calculators are in working condition.

4. Candidates are permitted to bring a set of spare batteries in the examination room.

5. No compensation will be given to candidates because of faulty calculators.

6. No help or advice is permitted on the use or repair of calculators during the examination.

7. Sharing calculators is not permitted in the examination room.

8. Instruction manuals, and external storage media are not permitted in the examination room.

9. Calculators with graphical display, data bank, dictionary or language translation are not allowed.

10. Calculators that have the capability of communication with any agency in or outside of the examination room are prohibited.

♦ SECTION 1: ALGEBRA, *SEQUENCES AND SERIES*

GENERAL OBJECTIVES

On completion of this Section, students should:

1. be confident in the manipulation of algebraic expressions and the solutions of equations and inequalities;

2. understand the difference between a sequence and a series;

3. recognise the differences between convergence and divergence of arithmetic and geometric *series; and,*

4. develop the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT
A. Algebra	
Students should be able to:	
1. perform operations of addition, subtraction, multiplication and division of polynomial and rational expressions;	Addition, subtraction, multiplication, division and factorization of algebraic expressions.
2. factorise polynomial expressions, of degree less than or equal to 4, leading to real linear factors;	Division of a polynomial of degree less than or equal to 4 by a linear or quadratic polynomial.
3. *determine the remainder when a polynomial is divided by a linear expression*;	*Applications of the* Remainder Theorem.
4. use the Factor Theorem to find factors; and,	Factor Theorem.
5. evaluate unknown coefficients using Factor Theorem.	

SPECIFIC OBJECTIVES	CONTENT
B. Quadratics	
Students should be able to:	
1. express the quadratic function $ax^2 + bx + c$ in the form $a(x + h)^2 + k$, where h and k are constants to be determined;	Completing the square.
2. determine maximum or minimum values and range of quadratic functions by completion of the square;	
3. sketch the graph of quadratic functions, including maximum or minimum points *and intercepts on the axes*;	Graphs of quadratic functions.
4. determine the nature of the roots of a quadratic equation;	*Use the discriminant* $D = b^2 - 4ac$
5. solve equations in x reducible to a quadratic equation;	For example, $x^4 - 6x^2 + 8 = 0$ $x - 2\sqrt{x} + 1 = 0$
6. use the relationship between the sums and products of the roots and the coefficients of $ax^2 + bx + c = 0$; and,	Applications of sums and products of the roots of quadratic equations.
7. solve two simultaneous equations in 2 unknowns in which one equation is linear and the other equation is *non-linear*.	Solution of equations (one linear and one *non-linear inclusive of quadratic and circle equations).* *For example:* (a) $y = ax + b$ $(x + f)^2 + (y + g)^2 = r^2$ (b) $px + q = y$ $ax^2 + bx + c = y$

SPECIFIC OBJECTIVES	CONTENT
C. Inequalities	
Students should be able to:	
1. *determine* the solution sets of quadratic inequalities using algebraic *or* graphical methods; and,	Quadratic inequalities in one unknown.
2. *determine* the solution sets of inequalities of the form $\frac{ax+b}{cx+d} > 0; \geq 0; < 0; \leq 0$ using algebraic *or* graphical methods.	*The use of set builder notation to represent the solution set.* Rational inequalities with linear factors.
D. Surds, Indices, and Logarithms	
Students should be able to:	
1. perform operations involving surds;	Addition, subtraction, multiplication and rationalisation of denominators of surds.
2. use the laws of indices to solve exponential equations with one unknown;	Equations reducible to linear and quadratic forms. *For example,* $2^{2x+1} - 3(2^x) - 2 = 0$
3. *use the fact that* $log_a b = c \Leftrightarrow a^c = b$ *where* $a \in \mathbb{N}$ *and* $b \in \cdot^{+}$;	The relationship between indices and logarithms.
4. simplify expressions by using the laws: (a) $\log_a(PQ) = \log_a P + \log_a Q$; (b) $\log_a \left(\frac{P}{Q}\right) = \log_a P - \log_a Q$; (c) $\log_a P^b = b \log_a P$; (d) $\log_a a = 1$; (e) $\log_a 1 = 0$;	Laws of logarithms. *Excluding the 'change of base' method.*

SECTION 1: ALGEBRA, *SEQUENCES AND SERIES* (cont'd)

SPECIFIC OBJECTIVES	CONTENT
Surds, Indices, and Logarithms (cont'd)	
Students should be able to:	
5. solve logarithmic equations;	For example, $\log_a(2x+5) - \log_a(3x-10) = \log_a(x-14)$
6. use logarithms to solve equations of the form $a^x = b$; and,	
7. apply logarithms to problems involving the transformation of a given relationship to linear form.	Linear reduction.
E. Sequences and Series	
Students should be able to:	
1. define a sequence of terms $\{a_n\}$ where *n* is a positive integer;	
2. write a specific term from the formula for the n^{th} term of a sequence;	
3. define a series, as the sum of the terms of a sequence;	Series as the sum of the terms of a sequence. *Use the summation* ($\sum$) *notation to include the rules associated.* *For example,* $\sum_{i=1}^{n} k\,a_i = k\sum_{i=1}^{n} a_i$
4. identify arithmetic and geometric series;	
5. *derive expressions for the* general terms and sums of finite arithmetic, and finite and infinite geometric series;	The sums of finite arithmetic, and finite and infinite geometric series.

SPECIFIC OBJECTIVES	CONTENT
Sequences and Series (cont'd)	
Students should be able to:	
6. show that all arithmetic series (except for zero common difference) are divergent, and that geometric series are convergent only if -1 < r < 1, where r is the common ratio;	*Examination of the formulae for both arithmetic series and geometric series.*
7. calculate the sum of arithmetic and geometric series to a given number of terms; and,	*Application of the concepts of the arithmetic and geometric series to solve real-world problems such as investments.*
8. find the sum of a convergent geometric series.	

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

Number Systems

1. Engage students in a review of the Number Systems before starting Algebra, Sequences and Series.

2. Engage students in the use of online applications such as Desmos or Geogebra *to explore the concepts in this section*.

3. Allow students to use and carry out the activities from the following websites which provide some useful classroom activities on Introducing logarithms as well as point to other websites.

   - http://www.jamestanton.com/wp-content/uploads/2012/03/Curriculum-Newsletter_April-2013.pdf

   - http://www.resourceaholic.com/2014/04/logs.html

   - https://www.stem.org.uk/elibrary/resource/32050

4. *As homework, have students watch the following video which provides an introduction to Sequences and Series. Engage students in a discussion around the concepts explained in the video.*

   - *http://www.mathcentre.ac.uk/types/motivating-mathematics/apgp/*

5. *Ask students to work in groups to make short presentations on*
   - *When a sequence becomes a series*
   - *Arithmetic vs Geometric sequences*
   - *Summing a sequence*

6. *This activity builds on students' knowledge of quadratic functions learnt in CSEC® Mathematics. Using Geogebra or other graphing tool, explore the properties of quadratic functions* $ax^2 + bx + c$, *to include what happens when the values of a, b, c are changed (two held constant, other variable changed, both positive and negative values), how they relate to the graph. Working in groups, have students respond to the following questions:*

   - *Is the graph symmetrical? Always? How can you determine the line/axis of symmetry? Is there a way to determine the axis of symmetry from the equation of the graph?*

   - *What is the significance of the point(s) where the graph crosses the x-axis? Does the graph always cross the x-axis? How many times does the graph cross the x-axis?*

   - *What is the significance of the point(s) where the graph crosses the y-axis? Does it always cross the y-axis? How many times would it cross the y-axis?*

♦ SECTION 2: COORDINATE GEOMETRY, VECTORS AND TRIGONOMETRY

GENERAL OBJECTIVES

On completion of this Section, students should:

1. develop the ability to represent and deal with points in the coordinate plane through the use of geometry and vectors;

2. develop the ability to manipulate and describe the behaviour of trigonometric functions;

3. develop skills to solve trigonometric equations;

4. develop skills to prove simple trigonometric identities; and,

5. develop the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT
A. Coordinate Geometry	
Students should be able to:	
1. *determine* the equation of a straight line;	*The equation of a straight line can be written in any form. For example,* $y = mx + c$ $ax + by + c = 0$
2. determine whether lines are parallel or mutually perpendicular using the gradients;	Relationships between the gradients of parallel and mutually perpendicular lines.
3. *determine* the point of intersection of two *straight* lines;	
4. write the equation of a circle;	The equation of the circle in the forms $(x + f)^2 + (y + g)^2 = r^2$ $x^2 + y^2 + 2fx + 2gy + c = 0$, where $f, g, c, r \in$ ▪
5. *determine* the centre and radius of a given circle;	

SPECIFIC OBJECTIVES	CONTENT
Coordinate Geometry (cont'd)	
Students should be able to:	
6. *determine* equations of tangents and normals at given points on circles; and,	Tangents and normals to the circle.
7. *determine* the point*(s)* of intersection of a *circle* with a straight line.	*Refer to Section 1B, Specific Objective E:7*
B. Vectors	
Students should be able to:	
1. *distinguish between scalar and vector quantities;*	*Distance and displacement, speed and velocity.*
2. express a vector in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ or $x\mathbf{i} + y\mathbf{j}$; $x, y \in R$;	Two-dimensional vectors and their geometric representations.
3. define equal vectors;	Equality of vectors.
4. (a) add vectors; and, (b) subtract vectors.	
5. multiply a vector by a scalar quantity;	
6. use unit vectors;	Unit vectors.
7. find displacement vectors;	Position and displacement vectors.
8. find the magnitude and direction of a vector;	Modulus and direction of a vector.
9. define the scalar product of two vectors: (a) in terms of their components; and, (b) in terms of their magnitudes and the angle between them.	Scalar (dot) product of 2 vectors.

SPECIFIC OBJECTIVES	CONTENT
Vectors (cont'd)	
Students should be able to:	
10. find the angle between two given vectors; and,	
11. apply properties of parallel and perpendicular vectors.	Problems involving parallel and perpendicular vectors.

C. Trigonometry ***(all angles will be assumed to be measured in radians unless otherwise stated)***

Students should be able to:	
1. define the radian;	
2. convert degrees to radians and radians to degrees;	
3. use the formulae for arc length $l = r\theta$ and sector area $A = ½\, r^2\theta$;	*Applications of arc length and sector area.*
4. evaluate sine, cosine and tangent for angles of any size given either in degrees or radians;	
5. evaluate the exact values of sine, cosine and tangent for $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2};$	*Include related angles such as* $\frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi$ ….
6. *draw the* graphs *of* the functions $\sin kx, \cos kx, \tan kx$, *where* k *is 1 or 2 for the range* $0 \leq x \leq 2\pi$;	*Basic properties of graphs of trigonometric functions, such as amplitude and periodicity.*

SPECIFIC OBJECTIVES	CONTENT
Trigonometry (cont'd)	
Students should be able to:	
7. derive the identity $cos^2\,\theta + sin^2\,\theta \equiv 1$;	
8. *use the identity* $\tan\theta \equiv \frac{sin\theta}{cos\theta}$;	
9. use the formulae for $\sin(A \pm B)$, $cos\,(A \pm B)$ and, $tan\,(A \pm B)$.	Compound-angle formulae.
10. derive the multiple angle identities for $sin\,2x, cos\,2x, tan\,2x$;	Double-angle formulae.
11. using Specific Objectives *7, 8 and 9* above prove simple identities; and,	
12. find solutions of simple trigonometric equations for the *range* $0 \leq \theta \leq 2\pi$, including those involving the use of $cos^2\,\theta + sin^2\theta \equiv 1$.	Solution of simple trigonometric equations including graphical interpretation but excluding general solution.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

Trigonometry

1. *Allow students to calculate the area of a segment and the area of triangle = ½ ab sin C.*

2. *Allow students to* derive the trigonometric identities and formulae where appropriate. However, students are not expected to know the proofs of the following trigonometric formulae:

   $\sin\,(A \pm B), \tan(A \pm B), \cos(A \pm B)$.

3. *Ask students to* use the equilateral and the isosceles right angled triangle to derive the exact values of sine, cosine and tangent of $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$.

4. *Allow students to* also derive the trigonometric functions sin *x* and cos *x* for angles *x* of any value (including negative values), using the coordinates of points on the unit circle.

5. *Conduct activities that will heighten students' awareness* of the relationships between the unit circle and its quadrants and the related angles (principal and secondary solutions).

6. *Encourage students to use the Excel plot to demonstrate the graphs of trigonometric functions.*

7. *Ask students to view the following website (*https://undergroundmathematics.org/ *and have students work in pairs or groups to match the equations for circles drawn on a pair of axes. The task itself can be found at:* https://undergroundmathematics.org/circles/teddy-bear *Specific teacher-related resources material can be found at:* https://undergroundmathematics.org/circles/teddy-bear-teacher-support/download/teddy-bear-teacher-support.pdf

8. *Use Geogebra or other graphing tools to investigate/explore the graphs of sine, cosine and tangent functions (for example, compare on same axes (i) sin x, sin 2x, sin 3x, 2 sin x, (ii) sin x, cos x, sin x + cos x, etc. Introduce students to the concepts such as periodicity and amplitude. Note area under the graphs.*

♦ SECTION 3: INTRODUCTORY CALCULUS

GENERAL OBJECTIVES

On completion of this Section, students should:

1. understand the relationships between the derivative of a function at a point and the behaviour of the function and its tangent at that point;

2. be confident in differentiating and integrating given functions;

3. understand the relationship between integration and differentiation;

4. know how to calculate areas and volumes using integration; and,

5. develop the ability to use concepts *such as kinematics* to model and solve real-world problems.

SPECIFIC OBJECTIVES	CONTENT
A. Differentiation	
Students should be able to:	
1. use the concept of the derivative at a point $x = c$ as the gradient of the tangent to the graph at $x = c$;	*Displacement-time graph.* *Velocity-time graph.* The gradient of a curve.
2. define the derivative at a point as a limit;	The derivative as a limit (intuitive approach).
3. use the $f'(x)$ and $\frac{dy}{dx}$ notation for the first derivative of $f(x)$;	
4. use $\frac{d}{dx}x^n = n\,x^{n-1}$ where n is any real number;	The derivative of x^n
5. use $\frac{d}{dx}\, sin\ ax = a\ cos\ ax$ and $\frac{d}{dx}\, cos\ ax = -a \sin ax$;	The derivatives of: $sin\ x$ and $cos\ x$; $sin\ ax$ and $cos\ ax$.

SECTION 3: INTRODUCTORY CALCULUS (cont'd)

SPECIFIC OBJECTIVES		CONTENT
Differentiation (cont'd)		
Students should be able to:		
6.	*use simple rules of differentiation to find derivatives of sums and multiples of function;*	Simple rules of *differentiation:* (a) $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$ where c is a constant; and, (b) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$. *Differentiation of simple polynomials and trigonometric functions involving sine and cosine only.*
7.	apply the chain rule in the differentiation of composite functions;	Function of a function, the chain rule.
8.	differentiate products and quotients of simple polynomials and trigonometric functions;	Product and quotient rules.
9.	*determine the equations of tangents and normals to curves;*	*Tangents and normals.*
10.	*calculate the second derivative, $f''(x)$;*	*Second derivatives of functions.*
11.	use the concept of the derivative as a rate of change;	*Real world applications to include:* (a) *Kinematics: rates of change of displacement and velocity where* $v = \frac{dx}{dt} = \dot{x}$; $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \ddot{x}$ *Where $x, \dot{x}, \ddot{x}$ represent displacement, velocity and acceleration respectively; and,* *(b) related rates of change for area and volume.*

SPECIFIC OBJECTIVES	CONTENT
Differentiation (cont'd)	
Students should be able to:	
12. *use the velocity time graph to determine acceleration;*	*Sketch the graph.* *Relate the gradient of the tangent at a point to acceleration.*
13. *use the concept of the stationary points;*	*Gradient equals zero.*
14. *determine the nature of stationary points (maxima/minima) by considering sign changes of the first derivative; and,*	*Application of stationary points to real-world problems involving maximum and minimum.* *Point(s) of inflexion not included.*
15. *use the sign of the second derivative to determine the nature of stationary points.*	
B. Integration	
Students should be able to:	
1. recognize integration as the reverse process of differentiation;	Anti-derivatives.
2. use the notation $\int f(x)dx$;	Indefinite integrals (concept and use).
3. show that the indefinite integral represents a family of functions which differ by constants;	
4. use simple rules of integration *to find integrals of sums and multiples of function;*	*Simple rules* of Integration. (a) $\int cf(x)\,dx = c\int f(x)\,dx$, where c is a constant; (b) $\int \{f(x) \pm g(x)\}\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

SECTION 3: INTRODUCTORY CALCULUS (cont'd)

SPECIFIC OBJECTIVES	CONTENT
Integration (cont'd)	
Students should be able to:	
5. integrate functions of the form $(ax \pm b)^n$ where a, b, n are real and $n \neq$-1;	Integration of polynomials.
6. integrate simple trigonometric functions;	Integration of $a\, sin\, px \pm b\, cos\, qx$, where a, b, p and q are constants.
7. compute definite integrals;	The definite integral: $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$ where $F(x)$ is the indefinite integral of $f(x)$.
8. formulate the equation of a curve given its gradient function and a point on the curve; and,	
9. apply integration to *find*:	
(a) the area of the region in the first quadrant bounded by a curve and the lines parallel to the *y*-axis;	
(b) volumes of revolution about the *x*-axis, for polynomials up to and including degree 2; and,	The region of the curve to be rotated must be in the first quadrant only.
(c) *displacement from velocity and velocity from acceleration.*	*Kinematics* $s = \int v\, dt$ $v = \int a\, dt$ *Variable motion of a particle.*

SECTION 3: INTRODUCTORY CALCULUS (cont'd)

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below.

Differentiation

1. *Students should be introduced to* the derivative as a limit. An intuitive approach is recommended at this stage by using diagrams and not first principles.

2. *Ask students to draw a tangent line at a particular point on a curve to calculate the value of the gradient at this point. Compare this value to the one obtained by differentiation.*

   A graphical explanation of $\frac{d}{dx}\,(\sin x) = \cos x$ would suffice.

3. *Students should be introduced to* the concept of increasing and decreasing functions. However, this will not be tested.

   Curve sketching using the differential calculus is not required.

4. *Allow students to use the first and second derivatives to determine maximum sales revenue, given the quadratic equation of the demand model.*

5. *By exploring examples, have students determine a rule for differentiating expressions of the type: c/x where c is a constant. Have students extend what they have discovered, to determine a rule for differentiation which covers expressions of the type: c/x^n, $n \in$ ▪ .*

   Have students compare and comment on differentiating expressions of these types via a conversion of the expression to cx^{-n} as an alternative to the use of the quotient rule.

The Area under the Graph of a Continuous Function

6. *Engage students in a class discussion on this topic*. Activities such as that which follows may be performed *in groups* to motivate the discussion.

   Example of classroom activity:

   Consider a triangle of area equal to $\frac{1}{2}$ unit^2, bounded by the graphs of y = x, y = 0 and x = 1.

   (a) Sketch the graphs and identify the triangular region enclosed.

   (b) Subdivide the interval [0, 1] into *n* equal subintervals.

   (c) Evaluate the sum, *s(n)*, of the areas of the inscribed rectangles and *S(n)*, of the circumscribed rectangles, erected on each subinterval.

   (d) By using different values of *n*, for example *n* = 5, 10, 15, 20, show that both *s(n)* and *S(n)* get closer to the area of the given region.

7. *Give students the opportunity to plan and carry out activities that will expose them to the information below.*

KINEMATICS

Definitions

Displacement *is the position of a point relative to a fixed origin O. It is a vector. The SI Unit is the metre (m). Other metric units are centimeter (cm), kilometer (km).*
Velocity *is the rate of change of displacement with respect to time. It is a* ***vector****. The SI Unit is* ***metre per second*** *(ms^{-1}). Other metric units include kmh^{-1}.*

Speed *is the magnitude of the velocity and is a scalar quantity.*

Uniform velocity *is the constant speed in a fixed direction.*

$$\text{Average velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{time taken}}$$

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

Acceleration *is the rate of change of velocity with respect to time. It is a* ***vector****. The SI Unit is* ***metre per second square*** *(ms^{-2}). Other metric units include kmh^{-2}.*

Negative acceleration *is also referred to as retardation.*

Uniform acceleration *is the constant acceleration in a fixed direction*

A particle is instantaneously at ***rest*** *when its velocity is zero.*
A particle reaches ***maximum velocity*** *when its acceleration is zero.*

GRAPHS IN KINEMATICS

A ***displacement-time*** *graph for a body moving in a straight line shows its displacement, x, from a fixed point on the line plotted against time, t. The* ***velocity****, v, of the body at time, t, is given by the* ***gradient*** *of the graph since* $\frac{dx}{dt} = v$.

SECTION 3: INTRODUCTORY CALCULUS (cont'd)

The **displacement-time** graph for a body moving with **constant velocity** is a **straight line**.
The velocity, v, of the body is given by the gradient of the line.

*The **displacement-time** graph for a body moving with **variable velocity** is a **curve**.*

The velocity at any time, t, may be estimated from the gradient of the tangent to the curve at that time. The average velocity between two times may be estimated from the gradient of the chord joining them.

***Velocity-time graph** for a body moving in a straight line shows its velocity, v, plotted against time, t.*

*The **acceleration**, a, of a body at time, t, is given by the **gradient** of the graph at t, since* $a = \frac{dv}{dt}$.

*The **displacement** in a time interval is given by the **area** under the **velocity-time** graph for that time interval.*

*The **velocity-time** graph for a body moving with **uniform acceleration** is a **straight line**. The acceleration of the body is given by the gradient of the line.*

*The **velocity-time** graph for a body moving with **variable acceleration** is a **curve**.*

Allow students to use the information and carry out the activities as outlined in the table below.

Information	***Activity-Group A***	***Activity-Group B***
A particle is moving with a constant acceleration which causes its speed to change from u to v in the time t .	• ***Draw a graph** showing the particle's change in speed.* • *Show that the equation of the straight line above is $v = u + at$.* • *Show that the distance travelled by the particle is* $x = ut + \frac{1}{2}at^2$	*Given $v = u + at$, where u, a, $\in$ ▪ , **use calculus** to show that* • $\frac{dv}{dt} = a.$ • $\int vdt$ *is* $x = ut + \frac{1}{2}at^{\,2}$
The velocity, v km/h, of a body after t hours is given as $v = 5t - t^2$	• *Draw the graph to represent the movement of the body during the first 5 hours.* • *Hence,* (i) *Determine the acceleration of the body at 3 hours.* (ii) *Determine the distance travelled by the particle during the first two hours.*	*Using **calculus**,* Determine the acceleration of the body at 3 hours. • *Determine the distance travelled by the particle during the first two hours.* • *Compare answers with Group A*

♦ SECTION 4: *PROBABILITY AND STATISTICS*

GENERAL OBJECTIVES

On completion of this Section, students should:

1. appreciate that data can be represented both graphically and numerically to initiate analysis;

2. understand the concept of probability; and,

3. appreciate that probability models can be used to describe real-world situations.

SPECIFIC OBJECTIVES	CONTENT
A. Data representation and Analysis	
Students should be able to:	
1. distinguish between types of data;	Qualitative and quantitative data, discrete and continuous data.
2. determine quartiles and percentiles from raw data, and grouped data;	*Percentiles.* *Range, interquartile range, semi-inter-quartile range.*
3. represent numerical data diagrammatically;	Stem-and-leaf diagrams and box and-whisker plots.
4. outline the relative advantages and disadvantages of stem-and-leaf diagrams and box-and-whisker plots in data analyses;	
5. interpret stem-and-leaf diagrams and box-and-whiskers plots;	
6. calculate measures of dispersion *using the given mean*; and,	*Variance and standard deviation of ungrouped and grouped data*.
7. *interpret* measures of central tendency and dispersion.	Mode, mean, median, range, interquartile range, semi-inter-quartile range, variance and standard deviation of ungrouped and grouped data.

SECTION 4: *PROBABILITY AND STATISTICS* (cont'd)

SPECIFIC OBJECTIVES	CONTENT
B. Probability Theory	
Students should be able to:	
1. distinguish among the terms experiment, outcome, sample space and event;	Concept of probability.
2. calculate the probability of event A, P(A), as the number of outcomes of A divided by the total number of possible outcomes, when all outcomes are equally likely and the sample space is finite;	Classical probability. Relative frequency as an estimate of probability.
3. use the basic laws of probability:	*Relate to Set Theory.*
(a) the sum of the probabilities of all the outcomes in a sample space is equal to one;	
(b) $0 \leq P(A) \leq 1$ for any event A; and,	
(c) $P(A') = 1 - P(A)$, where $P(A')$ is the probability that event A does not occur.	
4. use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to calculate probabilities;	The addition rule. *Relate to Set Theory.*
5. identify mutually exclusive events A and B such that $P(A \cap B) = 0$;	Mutually exclusive events. *Relate to Set Theory.*
6. calculate the conditional probability $P(A\|B)$ where $P(A\|B) = \frac{P(A \cap B)}{P(B)}$;	Conditional probability. *Relate to Set Theory.*

SECTION 4: *PROBABILITY AND STATISTICS* (cont'd)

SPECIFIC OBJECTIVES	CONTENT
Probability Theory (cont'd)	
Students should be able to:	
7. identify independent events;	Independent events. *Relate to Set Theory.*
8. use the property $P(A \cap B) = P(A) \times P(B)$ or $P(A\|B) = P(A)$ *where* A *and* B *are independent events;*	
9. construct possibility space diagrams, tree diagrams and Venn diagrams to solve problems involving probability; *and,*	Possibility space diagrams, tree diagrams *(limited to two initial branches)* and Venn diagrams *(limited to two sets)*.
10. use possibility space diagrams, tree diagrams and Venn diagrams to solve problems involving probability.	

<u>Suggested Teaching and Learning Activities</u>

To facilitate students' attainment of the objectives of this Section, teachers are advised to engage students in the teaching and learning activities listed below. Whenever possible, class discussions and presentations should be encouraged.

Probability

1. *Place students in groups and ask them to consider the four scenarios given below. This should be followed by a class discussion on the rules of Probability.*

   (a) *Play games such as Monopoly, Snakes and Ladder and Ludo.* Find the probability that the sum of the dots on the uppermost faces of the dice is 6.

   (b) *Play a card game. Find the probability of selecting a specific card, for example, a 'Queen'.*

   (c) An insurance salesman visits a household. What is the probability that he will be successful in selling a policy?

   (d) A hurricane is situated 500 km east of Barbados. What is the probability that it will hit the island?

These *four* scenarios are very different for the calculation of probability. In '1 and *'2'*, the probability is calculated as the number of successful outcomes divided by the total possible number of outcomes. In this classical approach, the probability assignments are based on equally likely outcomes and the entire sample space is known from the start.

The situation in *'c'* is no longer as well determined as in *'a' and 'b'*. It is necessary to obtain historical data for the salesman in question and estimate the required probability by dividing the number of successful sales by the total number of households visited. This frequency approach still relies on the existence of data and its applications are more realistic than those of the classical methodology.

For *'d'* it is very unclear that a probability can be assigned. Historical data is most likely unavailable or insufficient for the frequency approach. The statistician might have to revert to informed educated guesses. This is quite permissible and reflects the analyst's prior opinion. This approach lends itself to a Bayesian methodology.

One should note that the rules of probability theory remain the same regardless of the method used to estimate the probability of events.

STATISTICS

2. *Organise a field trip and allow students to work in groups to collect data using questionnaires, for example, on the height of students in the form/grade level; colours of candy in a packet; types/colours of vehicles in a car park.*

The data should be appropriately used to do any of the following:

(a) *identify different types of data;*
(b) *draw a stem and leaf diagram;*
(c) *plot box and whisker;*
(d) *perform calculations, for example, measures of spread; and,*
(e) *interpret and analyse data.*

♦ GUIDELINES FOR THE SCHOOL-BASED ASSESSMENT

RATIONALE

School-Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills and attitudes that are associated with the subject. The activities for the School-Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus. *Students are encouraged to work in groups.*

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School-Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School-Based Assessment. The guidelines provided for the assessment of these assignments. They are also intended to assist teachers in awarding marks that are reliable estimates of the achievement of students in the School-Based Assessment component of the course. In order to ensure that the scores awarded by teachers are in line with the **CXC®** standards, the Council undertakes the moderation of a sample of the School-Based Assessment assignments marked by each teacher.

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the student at various stages of the experience. This helps to build the self-confidence of students as they proceed with their studies. School-Based Assessment also facilitates the development of the critical skills and abilities that are emphasised by this **CSEC®** subject and enhances the validity of the examination on which candidate performance is reported. School-Based Assessment, therefore, makes a significant and unique contribution to the development of relevant skills and the testing and rewarding of students for the development of those skills.

The **Caribbean Examinations Council®** seeks to ensure that the School-Based Assessment scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

Assignment

The School-Based Assessment consists of one project to be marked by the teacher in accordance with **CXC®** guidelines.

There are two types of project.

Project A is based on applying mathematical concepts, skills and procedures from any topic (s) in order to understand, describe or explain a real world phenomenon. The project is theory based and no data collection is required.

Project B is based on applying mathematical concepts, skills and procedures from any topic (s) in order to understand, describe or explain a real world phenomenon. The project is experiment based and involves the collection of data.

Candidates should complete one project, either Project A or Project B.

Role of the Teacher

The role of teacher is to:

1. *Explore different possible* projects for the School-Based Assessment.

2. Provide guidance throughout the life of the projects. The teacher should work with candidates to develop a project management chart with definite time lines for achieving clearly identified objectives, from project inception to project completion.

3. Guide the candidate through the SBA by helping to clarify the problem or by discussing possible approaches to solving the problem. Teachers, while giving guidance, should guard against providing a complete solution to the problem for the candidate or prescribing a specific format that should be followed.

4. Ensure that the project is developed as a continuous exercise that occurs during scheduled class hours as well as outside class times.

5. At a time to be determined by the teacher the relevant component will be assessed and the marks recorded. Hard copies of the completed documents should be kept by both the teacher and student. The teacher should use the mark scheme provided by **CXC®** and include any comments pertinent to the conduct of the assessment.

♦ ASSESSMENT *CRITERIA*

Candidates should complete one project, either Project A or Project B. *Candidates have the option of working in smaller groups to complete the SBA.*

Project A

The following are the guidelines for assessing this project.

1. Candidates pursuing Additional Mathematics can complete a project which will be based on applying the mathematical concepts, skills and procedures from any topic (s) contained in any of the sections or their combinations in order to understand, describe or explain a real world phenomenon.

   The project will be presented in the form of a report and will have the following parts.

   (a) A statement of the problem

   A real-world problem in Mathematics chosen from any subject or discipline such as Science, Business or the Arts. The student must solve the problem using Specific Objectives completed in the course. This solution will involve either a proof or direct application of the concepts. (Data collection is not usually required for this project. Any necessary data should be given in the problem statement.)

   (b) Identification of important elements of the problem.

   (c) Mathematical formulation of the problem.

   (d) Selection of a method to solve the problem. This should involve use of Specific Objectives.

   (e) Solution of the resulting mathematical problem.

   (f) Interpretation of the solution in related real world context.

   (g) Conclusions reached.

2. The project will be graded out of a total of 30 marks. *The following are important considerations for grading the SBA.*

   (a) Clarity of the title of the real world problem being studied.

   (b) Scope/purpose of the problem.

   (c) Mathematical formulation of the problem.

   (d) The problem solution including explanation at each step.

   (e) Applications of the solution or proof to the given real world problem.

   (f) Discussion of findings and conclusion statement (this should include suggestions for further analysis of the topic).

   (g) Presentation (including an optional oral question and answer time with the teacher).

Assessing Project A

The project will be graded out *of* a total of 30 marks and marks will be allocated to each task as outlined below.

1. **Project Title**
   - Title is clear and concise, and relates to real world problem (2) [2]
   - *Title is mostly clear, concise and some what relates to real world problem* (1)

2. **Purpose of Project/Problem Statement**
   - Purpose is clearly stated and is appropriate in level of difficulty (2) [2]
   - Purpose clearly stated but is not appropriate in level of difficulty (1)

3. **Mathematical Formulation**
   - Identifies all the important elements of the problem and shows complete understanding of the relationships among elements (2) [6]
   - Shows complete understanding of the problem's mathematical concepts and principles (1)
   - Uses appropriate mathematical terminology and notations to model the problem mathematically (1)
   - Uses appropriate Mathematical model/methods (2)

4. **The Problem Solution**
   - Assumptions are clearly stated (1) [11]
   - Proofs are well established (1)
   - Diagrams are appropriate and clearly labelled (2)
   - Explanations are sufficient and clearly expressed (2)
   - Theorems are appropriate and Formulae are relevant to the solution (2)
   - Calculations are precise without errors (2)
   - Solution is clearly stated (1)

5. **Application of Solution**
   - Applies the solution or proof to the given real world problem (2) [3]
   - Shows that solution of proof to given problem is valid (1)

6. **Discussion of Findings/Conclusion**
   - Discussion is worthwhile (2) [4]
   - Conclusion is valid and useful (1)
   - Suggestions for future analysis in related areas are incorporated. (1)

7. **Overall Presentation**
   - Presentation is clear and communicates information in a logical way using correct grammar, mathematical jargon and symbols. (1) [2]
   - Communicates information in a logical way using correct grammar, mathematical jargon and symbols some of the time. (1)

Total 30 marks

PROJECT A – EXEMPLAR

Project Title

An investigation to determine the optimal surface area of a square-based cuboid and a cylinder of equal volume.

Purpose of Project

For a business to make a maximum profit from a product there needs to be ways it can reduce on the amount of material used as the less material used will decrease the amount of money spent making the product and the more profit will be made. In this case a comparison will be made between a square-based cuboid container and a cylindrical container that will hold a volume of 500cm^3 of orange juice. To determine the surface areas of the containers a variety of widths (for the square-based cuboid container) and diameters (for the cylindrical container) will be used. Knowing the volumes, and the widths and diameters of containers, the heights of the containers with its corresponding width or diameter can be found. Having found the heights of the containers, the optimum surface *areas can* be found using the dimensions of the containers. When the surface area is found, the comparison will be made to determine which container to use and the dimensions of that container.

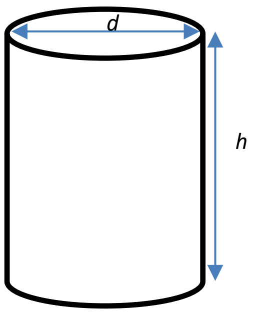
Another method that will be used is finding the differential of the surface area with respects to height to find the height which will produce the smallest surface area for that volume. The dimensions which correspond with the height will be found, and the surface area determined. A comparison of the values found will be made with the values of the investigation prior to it, to determine if the experiments were valid and to ascertain which one gave more appropriate values to suit the main purpose. The variables needed to carry out this investigation are *h, d, r, w,* π, *V* and *A* where *h* is the height, *d* is the diameter, *r* is the radius, *w* is the width, π is pi, *V* is the volume, and *A* is the area.

Mathematical *Formulation*

Volume is the amount of space occupied by an object.
Area is the measure of how much space there is on a flat surface.
Surface area is the total area of a surface of a three dimensional (3D) figure.
Height is the distance between the top and the bottom of an object.
Diameter is a chord that passes through the centre of a circle.
Radius is the distance from the centre of a circle, to any point on its circumference.
Width is the distance of an object from one side to the other, usually the horizontal distance.

Radius = $\dfrac{Diameter}{2}$

Cylinder



The volume of a cylinder is the product of the area of the base, and the height of the object. The base of the cylinder is a circle and the area is πr^2. The height of the cylinder is h. The product will result in the volume being $\pi r^2 h$.

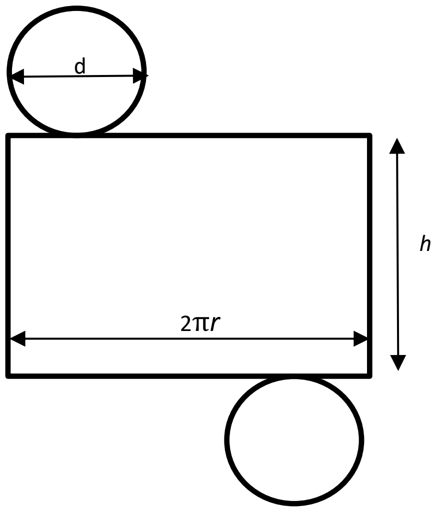
$Volume\ of\ a\ cylinder = \pi r^2 h$

To find the formula for height it was made the subject of the formula in the volume equation through the process of transposition (changing positions).

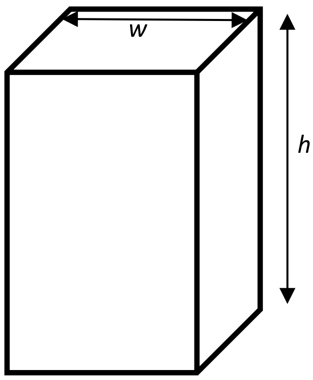
Height = $\frac{Volume}{\pi r^2}$

To calculate the total surface area of a *closed* cylinder, the sum of the areas of its faces needs to be found. If the *closed* cylinder is pulled apart two circles and a rectangle can be distinctly seen. The area of a circle is πr^2. Since there are two, it should be multiplied by 2 to obtain $2\pi r^2$. This will then be added to the area of the rectangle, length × width. In this case the length will be the circumference of the circle, $2\pi r$ and the width, the height, h. This will then result in it being $2\pi rh$. Having these two formulas the total surface area can be seen to be $2\pi rh + 2\pi r^2$.

*Total s*urface area of a *closed* cylinder = $2\pi rh + 2\pi r^2$



Square-Based Cuboid



Since the volume of a cuboid is the product of the area of the base and the height of the object. The area of a square is w^2 and the height of the cuboid is h which will result in the volume being w^2h.

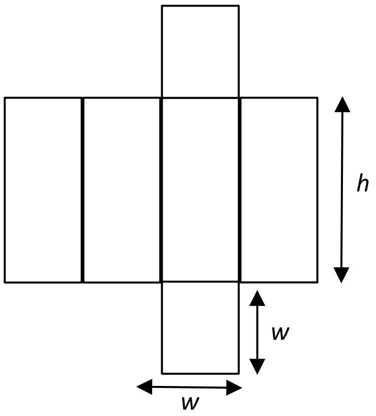
Volume of a square-based cuboid = w^2h

To find the formula for height, it was made the subject of the formula in the volume equation through the process of transposition (changing positions).

Height = $\dfrac{Volume}{w^2}$

To calculate the total surface area of a square-based cuboid, the sum of the areas *of its faces* need to be found. If the cuboid is pulled apart two squares and four rectangles can be distinctly seen. The area of a square is w^2. Since there are two squares it should be multiplied by 2 to get $2w^2$. This will then be added to the area of the rectangle, length × width. In this case, the length will be the height of the cuboid, h. This will then result in it being wh. Since there are four rectangles it should be multiplied by 4 to get $4wh$. Having these two formulae, the total surface area can be seen to be $2w^2 + 4wh$.

Total Surface Area of a square-based cuboid = $2w^2 + 4wh$



Solution of Problem

Cylinders

Above, the formulae used to calculate the height and surface area of the cylindrical container can be seen. Having the volume and diameter, the height can be calculated for any given case. Having found the values of height, the formula for the surface area of a cylinder was used to determine the surface area of the cylindrical container.

For example, to find the height of the cylinder when the diameter is equal to 10 cm:

$$h = \frac{v}{\pi r^2}$$

$$h = \frac{500}{\pi \times 5 \times 5} \qquad h = 6.37\text{cm}$$

To find the surface area of the cylinder when the diameter is equal 10 cm:

$$\boldsymbol{A = (\pi \times d \times h) + (2\pi r^2)}$$
$$A = (10 \times 6.37 \times \pi) + (\pi \times 5 \times 5 \times 2)$$
$$A = 357.1 \text{ cm}^2$$

Various diameters were used and the heights and surfaces areas were calculated in the same way. The results of these calculations are given in the table below.

Cylinder

Volume/cm^3	Diameter/cm^3	Height/cm	Surface Area/cm^2
500	5	25.46	439.3
500	10	6.37	357.1
500	15	2.83	486.8
500	20	1.59	728.3
500	25	1.02	1061.7

Square-Based Cuboids

The formulae used to calculate the height and surface area of the square-based cuboid container can be found above. Having the volume and width, the height can be calculated for any given case. Having found the values of height, the formula for the surface area of a square-based cuboid was used to determine the surface area of the square-based cuboid container.

For example, to find the height of a square-based cuboid when the width is 10 cm:

$$h = \frac{v}{w^2}$$
$$h = \frac{500}{10^2}$$
$$h = 5cm$$

To find the surface area of a square-based cuboid when the width is 10 cm:

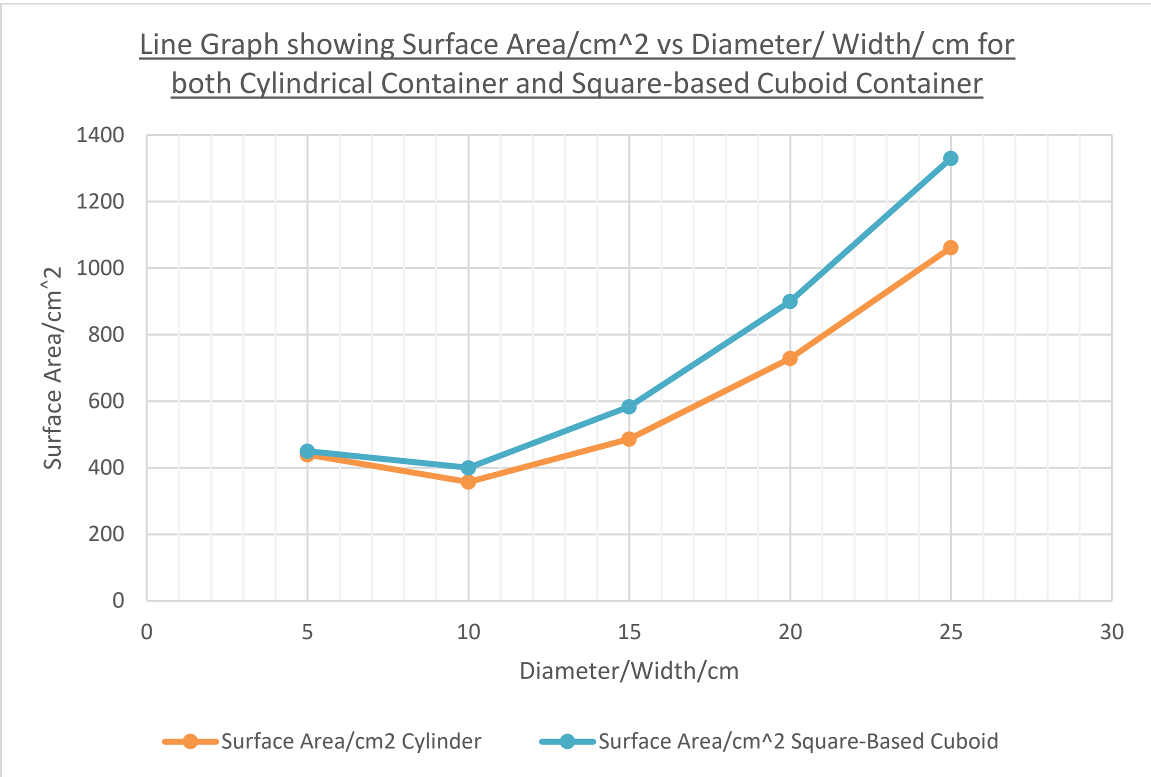
$$A = (2w^2) + (4wh)$$
$$A = (2 \times 10 \times 10) + (4 \times 10 \times 5)$$
$$A = 200 + 200$$
$$A = 400 \text{ cm}^2$$

Various widths were used and the heights and surfaces areas were calculated in the same way. The results of these calculations are given in the table below.

Square-Based Cuboid

Volume/cm^3	Width/cm	Height/cm	Surface Area/cm^2
500	5	20.00	450.0
500	10	400.00	400.0
500	15	583.83	583.33
500	20	900.00	900.00
500	25	1330.00	1330.00

A graph of surface area/cm^2 versus diameter/cm for the cylindrical container, and surface area/cm^2 versus width/cm for the square-based cuboid, can be plotted from the tables to show the comparison between the surface areas for both the cylindrical and square-based cuboid containers with the same width.



Another way the minimum surface area can be found is by equating the derivative of the area with respects to height (dA/dh) to zero (0) as shown below.

Cylinder

$$A = 2\pi rh + 2\pi r^2$$

$$V = \pi r^2 h$$

To reduce the number of variables in the equation for area, transpose the equation for volume to make r the subject of the equation.

$$\frac{v}{\pi h} = r^2$$

$$r = \sqrt{\frac{v}{\pi h}}$$

Substitute $\sqrt{\frac{v}{\pi h}}$ for r in the equation for A.

$$A = 2\pi h \times \sqrt{\frac{V}{\pi h}} + \frac{2V}{h}$$
$$A = 2 \times \sqrt{V\pi h} + 2Vh^{-1}$$

Derive A with respects to h.

$$\frac{dA}{dh} = \sqrt{\pi V} \times h^{\frac{-1}{2}} - 2Vh^{-2}$$

$$\frac{dA}{dh} = \frac{\sqrt{\pi v}}{\sqrt{h}} - \frac{2v}{h^2}$$

To find a value of h to give a minimum value of A let $\frac{dA}{dh} = 0$

$$\frac{\sqrt{\pi v}}{\sqrt{h}} - \frac{2v}{h^2} = 0$$

$$\frac{\sqrt{\pi v}\, h^{\frac{3}{2}} - 2v}{h^2} = 0$$

$$\sqrt{\pi V}\, h^{\frac{3}{2}} - 2V = 0$$

$$\sqrt{\pi V}\, h^{\frac{3}{2}} = 2V$$

$$h^{\frac{3}{2}} = \frac{2v}{\sqrt{\pi v}}$$

$$(h^{\frac{3}{2}})^2 = \left(\frac{2v}{\sqrt{\pi v}}\right)^2$$

$$h^3 = \frac{4v}{\pi}$$

$$h = \sqrt[3]{\frac{4v}{\pi}}$$

Substitute 500 for V in the equation above.

$$h = \sqrt[3]{\frac{4 \times 500}{\pi}}$$

$$h = 8.60 \text{ cm}$$

Square-Based Cuboid

$A = 2w^2 + 4wh$
$V = w^2h$

To reduce the number of variables in the equation for area, transpose the equation for volume to make w the subject of the equation.

$W = \sqrt{\frac{v}{h}}$

Substitute $\sqrt{\frac{v}{h}}$ for w in the equation for A

$$A = 2\frac{v}{h} + 4\sqrt{\frac{v}{h}} \times h$$

$$A = 2Vh^{-1} + 4\sqrt{v}\,h^{\frac{1}{2}}$$

Derive A with respects to h.

$$\frac{dA}{dh} = -2Vh^{-2} \quad + 2\sqrt{v}\,h^{\frac{-1}{2}}$$

To find the value of h to give a minimum value of A let $\frac{dA}{dh} = 0$

$$\frac{2\sqrt{V}}{\sqrt{h}} - \frac{2v}{h^2} = 0$$

$$\frac{2\sqrt{v}\,h^{\frac{3}{2}} - 2v}{h^2} = 0$$

$$2\sqrt{V}h^{\frac{3}{2}} - 2V = 0$$

$$h^{\frac{3}{2}} = \frac{2v}{2\sqrt{v}}$$

$$h^{\frac{3}{2}} = \sqrt{V}$$

$$h^3 = V$$

$$h = \sqrt[3]{V}$$

Substitute 500 for V in the equation above.

$$h = \sqrt[3]{500}$$

$$h = 7.94 \text{ cm}$$

Application of Solution

Cylinder
When h is 8.60 cm

$$V = \pi r^2 h$$

$$r^2 = \frac{v}{\pi h}$$

$$r = \sqrt{\frac{v}{\pi h}}$$

$$r = \sqrt{\frac{500}{\pi \times 8.60}}$$

$$r = 4.30 \text{ cm}$$

$$\therefore d = 2 \times 4.30$$

$$d = 8.60 \text{ cm}$$

When radius is 4.30 cm

$$A = 2\pi rh + 2\pi r^2$$
$$A = (8.60 \times \pi \times 8.60) + (2 \times \pi \times 4.30 \times 4.30)$$
$$A = 348.5 \text{ cm}^2$$

Square-Based Cuboid

When h is 7.94

$$V = w^2 h$$
$$w^2 = \frac{v}{v}$$
$$w = \sqrt{\frac{v}{h}}$$

$$w = \sqrt{\frac{500}{7.94}}$$

$w =$ 7.94 cm

When w is 7.94 and h is 7.94

$$A = 2w^2 + 4wh$$
$$A = 2(7.94)^2 + 4(7.94)(7.94)$$
$$A = 378.3 \text{ cm}^2$$

The values for height found for the cylinder and square-based cuboid by the derivative of area with respect to height produced have values similar to those found in the prior investigation. The values found were more precise *than* the ones before because *they gave definite* values for heights which *led to dimensions* which produced surface areas smaller than all the surface areas in the tables from the prior investigation.

Discussion

It is seen that in each case, the surface area of the cylindrical container is smaller than that of the square-based cuboid container in both investigations. This is so because the surface area of a square-based cuboid container is based mainly on the width of the base when compared to that of the cylindrical container which is based mainly on the radius of the base which is half the size of the diameter. Since both containers will hold a maximum of 500 cm^3, it will be more cost efficient if the container with the smallest surface area is used as the container for the juice. In this case, it will be a cylindrical container with a diameter of 8.60 cm, a height of 8.60 cm and a surface area of 348.5 cm^2. The container with the smallest surface area is used because the smaller the surface area the less amount of material will be used; the less amount of money it will cost, hence the more cost effective it will be. The cheaper the product, the more people will buy it, and the more money the business selling it will make.

Conclusion

For a business to make a maximum profit it should use a cylindrical container with a diameter of 8.60 cm and a height of 8.60 cm. This container will have a surface area of 348.5 cm^2 and will hold a volume of 500 cm^3.

For future analysis the equations $V = \pi r^2 h, A = 2 \times \sqrt{\pi V h} + 2Vh^{-1}$ and $h = \sqrt[3]{\frac{4v}{\pi}}$ can be used to find the volume, area and minimum value of h respectively of a cylinder by an individual who needs a cylindrical container for any specific purpose. By having these equations, the smallest amount of material needed to complete the cylinder can be found which can reduce wastage of material and also save money that would have been used to purchase material.

The equations $V = w^2 h, A = 2\frac{v}{h} + 4\sqrt{\frac{v}{h}} \times h$ and $h = \sqrt[3]{V} =$ can be used to find the volume, area and minimum value of h respectively of a square-based cuboid by an individual who needs a cuboid container for any specific purpose. By having these equations, the smallest amount of material needed to complete the cuboid can be found which can reduce wastage of materials and also save money that would have been used to purchase unnecessary material.

Project B

The following are guidelines for assessing this project.

1. Each candidate pursuing Additional Mathematics can complete a project which will be based on applying the mathematical concepts, skills and procedures from any topic(s) in order to understand, describe or explain a real world phenomenon. This project is experiment based and involves the collection of data.

   The project will be presented in the form of a report and will have the following parts:

   (a) A statement of the problem. A real world problem in Mathematics chosen from any subject or discipline such as Science, Business or the Arts. The student must solve the problem using Specific Objectives completed in the course. This solution will involve data collection which is required for this project.

(b) Identification of important elements of the problem.

(c) Formulation of a systematic strategy for representing the problem.

(d) Data collection appropriate for solving the problem.

(e) An analysis of the data, information and measurements.

(f) Solution of the resulting mathematical problem.

(g) Conclusions reached.

2. The project will be graded out of a total of 30 marks. The following are important considerations for grades the SBA.

(a) Clarity of the title of the real world problem being studied.

(b) Scope/purpose of the problem.

(c) Method of data collection.

(d) Presentation of data.

(e) Mathematical knowledge/analysis of data.

(f) Discussion of findings/conclusions.

(g) Presentation.

ASSESSING PROJECT B

The project will be graded out of a total of 30 marks and marks will be allocated to each task as outlined below.

Project Descriptors

1. **Project Title** [2]

- Title is clear and concise, and relates to real world problem (2)

2. **Purpose of Project** [2]

- Purpose is clearly stated and is appropriate in level of difficulty (2)

3. **Method of Data Collection** [4]

- Data collection method clearly described (1)
- Data collection method is appropriate and without flaws (2)
- Appropriate variables identified (1)

4. **Presentation of Data** [5]

- At least one table and one graph/chart used as relates to the syllabus (1)
- Graphs/charts correctly constructed (1)
- Tables, graphs /charts are clearly labelled and systematic (2)
- Statistics/mathematical symbols used appropriately (1)

5. **Mathematical Knowledge/Analysis of Data** [8]

- Appropriate use of mathematical concepts demonstrated (objectives related to Additional Mathematics) (3)
- Calculations are precise without errors (2)
- Some analysis attempted (1)
- Analysis is coherent (1)
- Analysis used a variety (two or more) approaches (1)

6. **Discussion of Findings/Conclusion** [7]

- Findings clearly identified (1)
- Discussion follows from data gathered/solution of problem (2)
- Limitations are clearly stated (1)
- Concluding statement based on findings and relates to purposes of project (2)
- Provides suggestions for future use (1)

7. **Overall Presentation** [2]

- Communicates information in a logical way using correct grammar, and spelling (1)
- Communicates information in a logical way using correct mathematical jargon and symbols (1)

Total 30 marks

PROJECT B – EXEMPLAR

Project Title
Simple experiments to determine the fairness of an ordinary game die.

Statement of Task
Classical probability states that the probability of any of the 6 faces of an ordinary cubical game die landing with a distinct face uppermost after being thrown is $\frac{1}{6}$. It is not unusual for one throwing an ordinary gaming die to observe that one particular face lands uppermost with more frequency than any of the other faces.

Is this sufficient reason for one to conclude that the die may be biased? It may be by chance that this phenomenon occurs, or, perhaps the manner in which the die is thrown has an effect on its outcome. An experiment of this nature may be affected by factors that vary because of the non-uniform manner in which it is conducted.

This project aims to carry out some simple experiments to determine whether or not some varying factors of the manner in throwing the die do in fact influence the outcomes.

Data Collection
An ordinary 6-face gaming die was chosen for this experiment. 120 throws were made for each experiment, using each of the following methods:

1. holding the die in the palm of the hand and shaking it around a few times before throwing it onto a varnished table top;

2. placing the die in a Styrofoam drinking cup, shaking it around a few times before throwing it onto a varnished table top;

3. placing the die in a smooth metal drinking cup, shaking it around a few times before throwing it onto a varnished table top;

4. holding the die in the palm of the hand and shaking it around a few times before throwing it onto a linen covered table top;

5. placing the die in a Styrofoam drinking cup, shaking it around a few times before throwing it onto a linen covered table top; and,

6. placing the die in a smooth metal drinking cup, shaking it around a few times before throwing it onto a linen covered table top.

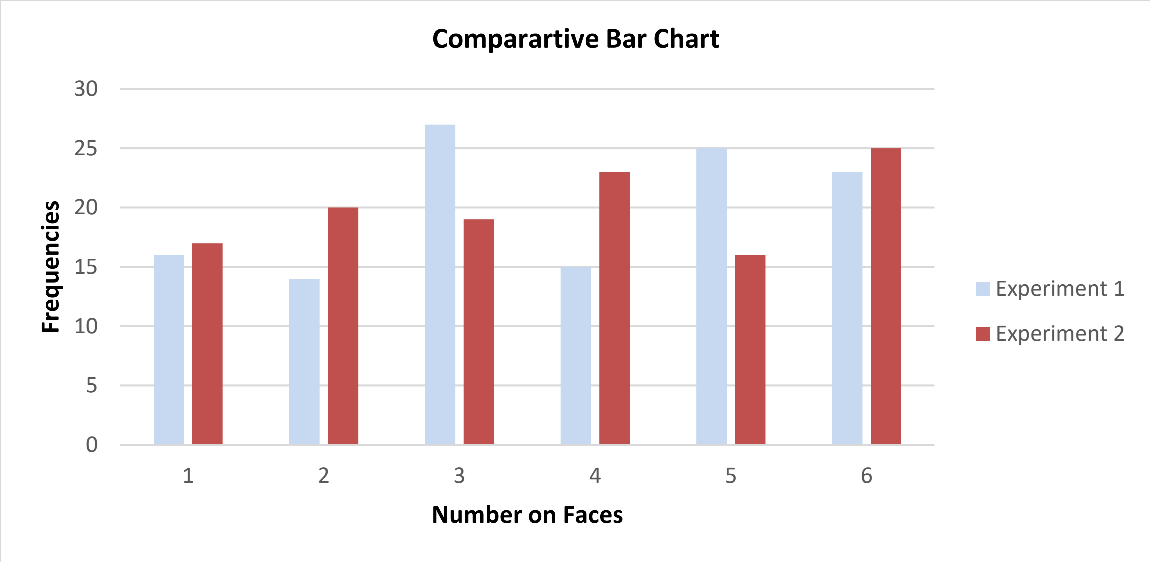
After each experiment the frequencies of the numbers landing uppermost were recorded in tabular form.

In each of these experiments, the number of times the die was shaken before throwing was not predetermined, nor was any other deliberate consideration applied in the subsequent throws. Every effort was taken to avoid bias in each of the experiments.

The following table shows the results of the experiments carried out.

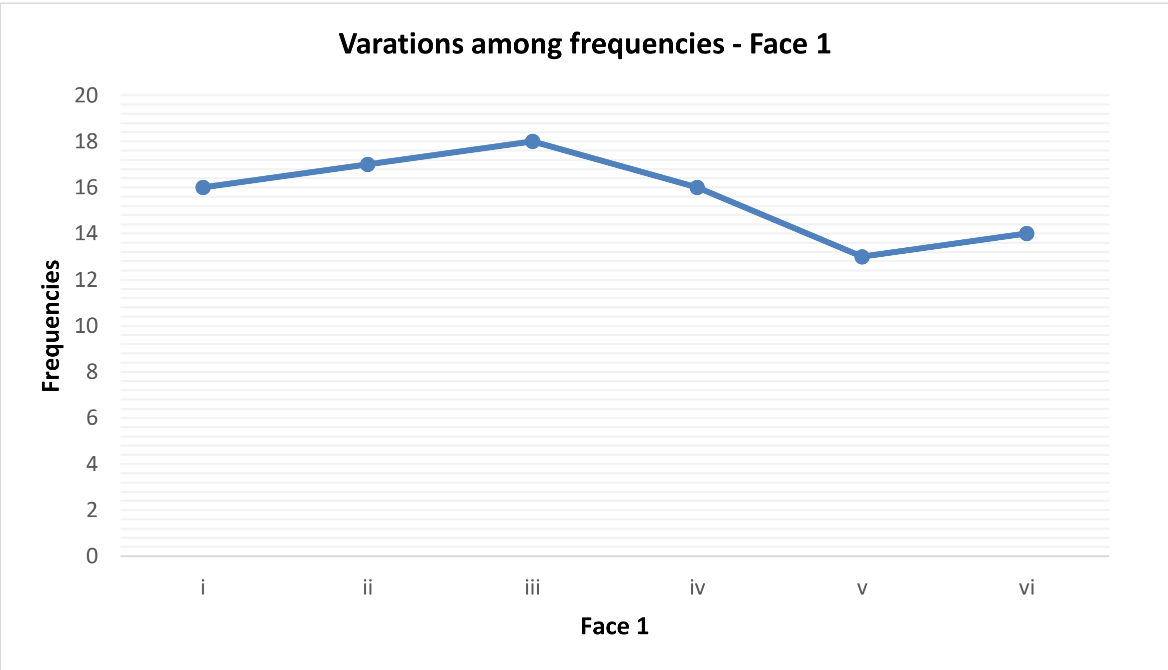
# on face	1	2	3	4	5	6
Frequencies - Exp (1)	16	14	27	15	25	23
Frequencies - Exp (2)	17	20	19	23	16	25
Frequencies - Exp (3)	18	25	20	19	25	13
Frequencies - Exp (4)	16	21	20	29	13	21
Frequencies - Exp (5)	13	20	27	18	19	23
Frequencies - Exp (6)	14	24	17	24	25	16
Total frequencies	94	124	130	128	123	121

Presentation of Data
The following comparative bar chart illustrates the variations of frequencies for obtaining the numbers 1 through 6 on the uppermost face for experiments (1) and (2).



Graphs to illustrate experiments (iii), (iv), (v) and (vi) may be shown as well.

The following line graph illustrates the variations among the frequencies for face 1.



Graphs for each of faces 2, 3, 4, 5, and 6 may be shown.

Mathematical Knowledge/Analysis of Data

Choosing to use the different methods for carrying out these experiments, as described in Data Collection, took into account that different conditions of the throws of the die may have significant influences in the outcomes of these throws. The size of the cups chosen may have a particular influence on these outcomes. The inside surfaces of the two types of cups chosen are also factors that may influence these outcomes. The number of times the die is tossed around in the palm of the hand and/or the number of times it is tossed around in the cups may influence these outcomes. The different coverings of the surface of the table top may also influence these outcomes.

In the absence of more in-depth and elaborate statistical techniques, these simple experiments were intended to give some idea of the theory of classical probability. The limiting relative frequency of an event over a long series of trials is the conceptual foundation of the frequency interpretation of probability. In this framework, it is assumed that as the length of the series increases without bound, the fraction of the experiments in which we observe the event will stabilize.

120 throws under each of the conditions selected should allow for simple comparison of the observed and theoretical frequencies.

Using the principle of relative probability, the following table shows the probability distribution for Experiment (1) and the theoretical probability of obtaining any of the faces numbered 1, 2, 3, 4, 5, 6 landing uppermost.

# on face	1	2	3	4	5	6
Relative probability	= 0.13	= 0.12	= 0.23	= 0.13	= 0.21	= 0.19
Theoretical probability	= 0.17	= 0.17	= 0.17	= 0.17	= 0.17	= 0.17

Comparisons of the differences of the observed and theoretical frequencies for 120 throws of the die under the conditions described should be considered as sufficient for an explanation of any significant variation in determining whether the die was biased in favour of any particular face. Barring any significant variation among the relative frequencies, it may be reasoned that the die is not biased.

The relative probabilities can also be calculated for Experiments (2) through (6)

Furthermore, we can combine the results of all six experiments to arrive at an overall probability for each face as shown in the table below:

# on face	1	2	3	4	5	6
Relative frequency	= 0.13	= 0.17	= 0.18	= 0.18	= 0.17	= 0.17

The above table clearly shows that the relative frequency of each face is close to the true probability (0.17) when the number of trials (720) is large. This is strong evidence to claim that the die is unbiased even though there were differences among the observed frequencies for the six experiments.

Further analysis must be taken in light of any limitations that the project may have. Considering the mean and standard deviation of each of these experiments, account may be taken of the size of the variations of the observed and theoretical values. This aspect may explain any significant variation from the expected mean and variance of these outcomes

The standard deviations for the frequencies of faces 1 through 6 for Experiments (i), (ii), (iii), (iv), (v) and (vi) are 1.67, 1.71, 1.62, 1.63, 1.63 and 1.60 respectively.

Except for Face #2 and to a lesser extent (Face #1), the variances among the outcomes do not appear to suggest significant differences in the results.

Conclusions

These experiments can be considered simplistic but reasonably effective for the purpose of determining bias in an ordinary gaming die. The number of throws, 120, may be considered sufficient for obtaining relative frequencies and relative probability for the experiments. Increasing the number of throws should result in observed frequencies very close to the theoretical frequencies.

Further statistical analyses can explain variations between the observed and theoretical results. These experiments may be refined by using other methods of throwing the die. Results can be compared for similarity among these results and for a reasonable conclusion about fairness of the die.

Procedures for Reporting and Submitting School-Based Assessment

1. Teachers are required to record the mark awarded to each candidate under the appropriate profile dimension on the mark sheet provided by **CXC®**. The completed mark sheets should be submitted to **CXC®** no later than April 30 of the year of the examination.

   Note: The school is advised to keep a copy of the project for each candidate as well as copies of the mark sheets.

2. Teachers will be required to submit to **CXC®** copies of the projects of a sample of candidates as indicated by **CXC®**. This sample will be re-marked by **CXC®** for moderation purposes.

Moderation of School-Based Assessment

The candidate's performance on the project will be moderated. The standard and range of marks awarded by the teacher will be adjusted where appropriate. However, the rank order assigned by the teacher will be adjusted only in special circumstances and then only after consideration of the data provided by the sample of marked projects submitted by the teacher and re-marked by **CXC®.**

Project A – Guidelines for Interpretation and Marking

Project Descriptors	Description	Award of marks	Total (30)
Project title	The title must clearly state **WHAT** the project is about. It can be a statement or a question. It must be both clear and concise.	Title is • Clear (1) • Concise (1)	2
Purpose	The purpose explains **WHY** the project is being done. What real-world problem is going to be solved?	Purpose is • Clearly stated (1) • Relates to real-world problem (1)	2
Mathematical formulation	This details **HOW** the problem described in the purpose is going to be solved.	• Identifies all the important elements in the problem (1) Shows clear understanding of the relationship between elements (1)	6
		• Shows understanding of the problem's mathematical concepts/ principles (1)	
		• Uses appropriate terminology and notations to model the problem mathematically (1)	
		• Use of appropriate mathematical model/methods chosen (1) Appropriate level of difficulty (1)	
The Problem solution	Following the instructions in the mathematical formulation, the problem solution is derived in this section of the project i.e. the problem is **SOLVED**.	• Assumptions taken are clearly stated (1)	11
		• Proofs are well established or stated (1)	
		• Diagrams are appropriate (1) Diagrams are clearly labelled (1)	
		• Explanations are sufficient, that is explanatory notes at each major step in the calculations are included (1) Explanations are clearly expressed, that is they are easy to follow (1)	
		• Theorem and/or formulae are relevant to the solution (1) Theorem and/or formulae are correctly applied (substitution) (1)	

		• Calculations contain no errors (2) • Calculations contain 1 - 2 errors (1) • Calculations contain more than 2 errors (0) • Typographical errors should not be penalised.	
		• Solution(s) is/are clearly stated (1)	
Application of solution	Here the solution derived is applied to the real-world context. How does the solution solve the problem?	• Some application attempted Applies the solution or proof to the given real-world problem correctly (2)	3
		• Shows that the solution or proof is valid. Validation can be achieved by substitution, use of an alternate method or proof or by comparison to some external data. (1)	
Discussion of Findings/ Conclusion	The discussion explores and interprets the solution obtained in light of the project's objectives. It also presents an evaluation of the particular method used. For example: Based on the results, how could the project be improved? What additional, future use or analysis is possible? What recommendations can be made?	• Discussion is coherent (can be followed easily, no ambiguity) (1) • Concise (without unnecessary redundancy and repetition) and relates to the purpose of the project (1)	4
		• Conclusion is valid (based on the study/ investigation) (1)	
		• Suggestions for future use and/or analysis in related areas (1)	
Overall Presentation	This is an assessment of the entire written report.	• Project report contains correct grammar and spelling most of the time (1)	2
		• Project report contains correct, mathematical jargon and symbols most of the time (1)	

Project B – Guidelines for Interpretation and Marking

Project Descriptors	**Description**	**Award of marks**	**Total (30)**
Project title	The title must clearly state **WHAT** the project is about. It can be a statement or a question. It must be both clear and concise.	Title is • Clear (1) • Concise (1)	2

Purpose	The purpose explains **WHY** the project is being done. What real-world problem is going to be investigated?	Purpose is • Clearly stated (1) • Related to real-world problem (1)	2
Method of Data Collection	This details **HOW** data will be collected, indicating the variables of the problem.	• Variables are identified (1) • Method is clearly described (1) • Type of sampling and sample size identified (1) Type of data identified (1)	4
Presentation of Data	In this section data collected is organised and visually represented in labelled tables, graphs/ charts. Appropriate statistical/mathematics symbols are used where appropriate.	• At least one table (1) • At least one graph and chart used from the Additional Mathematics syllabus. (For example, Select from, stem-and-leaf diagrams; box-and-whisker plots; probability space diagrams; probability tree diagrams) (1) • Tables, graphs /charts are clearly labelled (axes, units and title/heading) (1) • Tables, graphs/charts are placed in some (rational) order (systematic) (1) • Statistical/mathematical symbols used appropriately in most instances. For example Standard deviation (σ), frequency(f), Mean ($\bar{x}$) (1)	5
Mathematical knowledge/Analysis	Here the data collected is analysed. This process involves both quantitative and qualitative approaches to evaluate the data in relation to the purpose of the study/ investigation.	• Appropriate use of statistical/mathematical concepts demonstrated relevant to the purpose of the study - At the level of Additional Mathematics (1) - Appropriate use of statistical/mathematical concepts (1) - Relevant to the purpose of the study (1) • Calculations contain no errors (2)	8

		- 1- 2 errors (1) - More than 2 errors (0) Typographical errors should not be penalised.	
		• Some analysis attempted (1)	
		• Analysis is coherent i.e. - Analysis is comprehensive (complete or detailed; Links made to table/ graphs/charts or a summary table) (1)	
		• Analysis used a variety (two or more) approaches (1)	
Discussion of findings/ Conclusion	The discussion explores and interprets the solution obtained in light of the project's objectives. It also presents an evaluation of the particular method used so limitations are identified. The conclusion provides a summary of the results/findings as it relates to the purpose of the project.	• Findings must be clearly identified within the discussion (1)	7
		• Discussion follows from data gathered/solution to problem: - Comprehensive discussion (Includes all or most important elements) (2) OR - Partial Discussion (Includes only some important elements but is relevant) (1) OR - Discussion does not follow from solution and/or irrelevant (0)	
		• Any limitations of the study are stated (1)	
		• Concluding statement - Based on findings (1) - Relates to purposes of project (1)	
		• Suggestions for future use and/or analysis in related areas (1)	
Overall Presentation	This is an assessment of the entire written report.	• Project report contains correct grammar and spelling most of the time (1)	2
		• Project report contains correct, mathematical jargon and symbols most of the time (1)	

♦ RESOURCES

The following is a list of books and other resources material that might be used for Additional Mathematics. The list is by no means exhaustive. Each student should have access to at least one text.

Ali, F. and Khan, S.	*Developing Mathematical Minds For CSEC Additional Mathematics. 1st Ed. San Fernando: Caribbean Educational Publishers, 2013.*
Talbert, J. F. and Heng, H. H.	Additional Mathematics – Pure and Applied. Singapore: Longman Publishers, 1991.
Toolsie Bsc, R	*Additional Mathematics: A Complete Course for CSEC. San Fernando: Caribbean Educational Publisher, 2003.*

Websites:

https://nrich.maths.org

https://www.geogebra.org/materials/

https://www.tes.com/teaching-resources/hub/secondary/mathematics/advanced-pure

https://www.mathsisfun.com/data/
http://www.mathcentre.ac.uk/

http://www.coolmath.com/

http://www.sosmath.com/wwwsites.html

https://ima.org.uk/case-studies/mathematics-matters/

https://ima.org.uk/4188/the-potential-and-challenges-for-mathematics-teaching-and-learning-in-the-digital-age/

♦ GLOSSARY

WORD	DEFINITION	NOTES
analyse	examine in detail	
annotate	add a brief note to a label	Simple phrase or a few words only.
apply	use knowledge/principles to solve problems	Make inferences/conclusions.
assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
classify	divide into groups according to observable characteristics	
comment	state opinion or view with supporting reasons	

WORD	DEFINITION	NOTES
compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.
deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	
define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
demonstrate	show; direct attention to...	
derive	to deduce, determine or extract fromdata by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
determine	find the value of a physical quantity	
design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analyzed and presented.

WORD	DEFINITION	NOTES
develop	expand or elaborate an idea or argument with supporting reasons	
diagram	simplified representation showing the relationship between components	
differentiate/distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
discuss	present reasoned argument; consider points both for and against; explain the relative merits of a case	
draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
estimate	make an approximate quantitative judgement	
evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.
explain	give reasons based on recall; account for	
find	locate a feature or obtain as from a graph	
formulate	devise a hypothesis	
identify	name or point out specific components or features	
illustrate	show clearly by using appropriate examples or diagrams, sketches	
interpret	explain the meaning of	

WORD	DEFINITION	NOTES
investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
justify	explain the correctness of	
label	add names to identify structures or parts indicated by pointers	
list	itemize without detail	
measure	take accurate quantitative readings using appropriate instruments	
Name	give only the name of	No additional information is required.
note	write down observations	
observe	pay attention to details which characterize a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
outline	give basic steps only	
plan	prepare to conduct an investigation	
predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables.
relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
sketch	make a simple freehand diagram showing relevant	

WORD	DEFINITION	NOTES
	proportions and any important details	
state	provide factual information in concise terms outlining explanations	
suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalization which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
use	apply knowledge/principles to solve problems	Make inferences/conclusions.

Western Zone Office
24 September 2018

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

“*”Barcode Area”*”
Front Page Bar Code

SPECIMEN

FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE

0	1	2	5	4	0	3	2

SUBJECT ADDITIONAL MATHEMATICS – Paper 032

PROFICIENCY GENERAL

REGISTRATION NUMBER

SCHOOL/CENTRE NUMBER					

NAME OF SCHOOL/CENTRE

CANDIDATE’S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH

D	D	M	M	Y	Y	Y	Y

SIGNATURE ____

“*”Barcode Area”*”
Current Bar Code

“*”Barcode Area”*”
Sequential Bar Code

DO NOT

WRITE ON

THIS PAGE

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

Paper 032 – General Proficiency

ALTERNATIVE TO SCHOOL-BASED ASSESSMENT

90 minutes

SPECIMEN PAPER

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of ONE compulsory question.
2. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. A list of formulae is provided on page 4 of this booklet.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Required Examination Materials

Electronic calculator (non programmable)
Geometry set
Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

“*”Barcode Area”*”
Sequential Bar Code

LIST OF FORMULAE

Arithmetic Series	$T_n = a + (n - 1)d \qquad S_n = \frac{n}{2}[2a + (n - 1)d]$
Geometric Series	$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_\infty = \frac{a}{1 - r}, \quad -1 < r < 1 \text{ or } \lvert r \rvert < 1$
Circle	$x^2 + y^2 + 2fx + 2gy + c = 0 \qquad (x + f)^2 + (y + g)^2 = r^2$
Vectors	$\hat{\mathbf{v}} = \frac{\mathbf{v}}{\lvert\mathbf{v}\rvert} \qquad \cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{\lvert\mathbf{a}\rvert \times \lvert\mathbf{b}\rvert} \qquad \lvert\mathbf{v}\rvert = \sqrt{(x^2 + y^2)}$ where $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$
Trigonometry	$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$
	$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$
	$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
Differentitaion	$\frac{\mathrm{d}}{\mathrm{d}x}(ax + b)^n = an(ax + b)^{n-1}$
	$\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$
	$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$
Statistics	$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \qquad S^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n} = \frac{\sum_{i=1}^{n} f_i x_i^2}{\sum_{i=1}^{n} f_i} - (\overline{x})^2$
Probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Kinematics	$v = \frac{dx}{dt}, \qquad a = \frac{d^1y}{dx^1} = \frac{dv}{dt} = \ddot{x}$

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

NOTHING HAS BEEN OMITTED.

1. Mr Clarke is a retiring Mathematician who has always tried to incorporate Mathematics into every area of his life. **Exactly two years ago**, Mr Clarke decided he should have a party for his retirement. His wife gave him a savings box and $50 to get him started on saving towards the event. Excited about the plans, Mr Clarke saved $70 in the second month, $90 in the third month and so on. He committed to saving in this manner until the end of the two years.

(a) Formulate the the appropriate mathematical equation and calculate the total amount of money that Mr Clarke has saved after two years.

(3 marks)

(b) Mr and Mrs Clarke have 3 children. It has always been Mr Clarke's desire for each of his children to have 3 children and for each generation of Clarkes to do the same.

(i) If Mr Clarke's wish comes true, formulate the appropriate equation and calculate the number of great-great-grandchildren he would have.

(4 marks)

(ii) Assuming that everyone is still alive, formulate and calculate the total number of relatives in the bloodline in the Clarke family, up to the great-great-grandchildren, inclusive of Mr and Mrs Clarke.

(3 marks)

(c) Mr Clarke has 9 grandchildren, 5 boys and 4 girls. Currently 30% of the boys and 40% of the girls in the Clarke family would be accepted into university. Of those attending university 20% of the boys and 30% of the girls will graduate with first class honours.

Given that a grandchild is chosen at random, find the probability that

(i) the child will be accepted into university
(ii) the child will graduate university with first class honours
(iii) the child is a boy, GIVEN that the child will NOT be accepted into university.

Construct a well-labelled tree diagram to show the information. Include all associated probabilities and the sample space.

(10 marks)

(d) A student was given directions to Mr Clarke's house. The journey contains 3 landmarks. The student drives from home accelerating uniformly at 0.2 ms^{-2} for 1 minute and continues at that speed for 20 minutes before coming to rest in 5 minutes. During the journey the student arrived at the second landmark after travelling 1.5 km.

Determine the time taken to arrive at the second landmark.

(10 marks)

Total 30 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No. ☐

DO NOT

WRITE ON

THIS PAGE

CANDIDATE'S RECEIPT

INSTRUCTIONS TO CANDIDATE:

1. Fill in all the information requested clearly in capital letters.

TEST CODE:

0	1	2	5	4	0	3	2

SUBJECT: ADDITIONAL MATHEMATICS – Paper 032

PROFICIENCY: GENERAL

REGISTRATION NUMBER:

	S	P	E	C	I	M	E	N	

FULL NAME: ____
(BLOCK LETTERS)

Signature: ____

Date: ____

2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.

3. Keep it in a safe place until you have received your results.

INSTRUCTION TO SUPERVISOR/INVIGILATOR:

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature: ______________________
Supervisor/Invigilator

Date: ______________________

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

ADDITIONAL MATHEMATICS

PAPER 032 – GENERAL PROFICIENCY

SPECIMEN PAPER

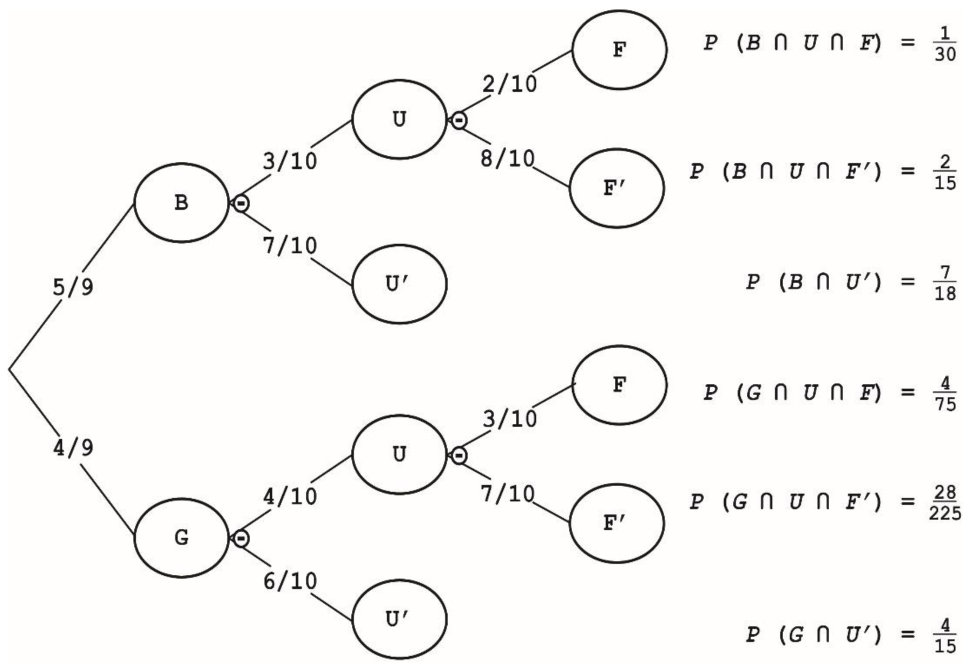
KEY AND MARK SCHEME

ADDITIONAL MATHEMATICS
PAPER 032 – GENERAL PROFICIENCY
KEY AND MARK SCHEME

Specific Objectives: Section 1: E2, E5, E7
Section 4: A3, B2-B5

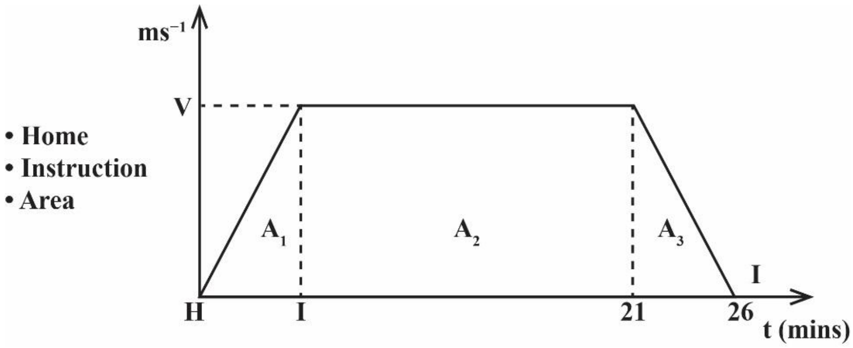
			CK	AK	R
(a)		This is an arithmetic progression			
		$a = 50$ $d = 20$ $n = 24$			1
		The total amount of money saved is the sum of the A.P			
		$S_n = \frac{n}{2}[2a + (n-1)d]$	1		
		$S_{24} = \frac{24}{2}[2(50) + (24-1)20]$			
		$S_{24} = 12[100 + (23)20]$			
		$S_{24} = \$6720$		1	
(b)	(i)	This is a geometric progression $a = 3$ $r = 3$			1
		The number of great-great-grandchildren is the fourth term in the G.P $(n = 4)$			1
		$T_n = ar^{n-1}$ $T_4 = 3(3)^3$	1		
		$T_4 = 81$		1	
		Mr. Clarke would have 81 great-great-grandchildren.			

ADDITIONAL MATHEMATICS
PAPER 032 – GENERAL PROFICIENCY
KEY AND MARK SCHEME

			CK	AK	R
(b)	(ii)	The number of offspring in the bloodline would be the sum of the G.P $S_n = \frac{a(r^n-1)}{r-1}$ $S_4 = \frac{3(3^4-1)}{3-1}$ $S_4 = \frac{3(80)}{2}$ $S_4 = 120$ However the total number of relatives also includes Mr. and Mrs. Clarke. Therefore, $total\ number\ of\ reltives\ = 120 + 2$ $= 122\ relatives$	1	1	1
(c)		Let B represent the event "the child is a boy" Let G represent the event "the child is a girl" Let U represent the event "the child will be accepted into university" Let F represent the event "the child will graduate with First Class Honors" 	1	2	1

			CK	AK	R
(c)		i) $P(U) = P(B \cap U)\ or\ P(G \cap U)$ $= \left(\frac{5}{9} \times \frac{3}{10}\right) + \left(\frac{4}{9} \times \frac{4}{10}\right)$ $= \frac{31}{90}$	1	1	
		ii) $P(F) = P(B \cap U \cap F)\ or\ P(G \cap U \cap F)$ $= \frac{1}{30} + \frac{4}{75}$ $= \frac{13}{150}$	1	1	
		iii) $P(B\|U') = \frac{P(B \cap U')}{P(U')}$ $= \frac{\frac{7}{18}}{\frac{7}{18} + \frac{4}{15}}$ $= \frac{7/18}{59/90}$ $= \frac{35}{59}$		1	1

ADDITIONAL MATHEMATICS
PAPER 032 – GENERAL PROFICIENCY
KEY AND MARK SCHEME

			CK	**AK**	**R**
(d)		Velocity-Time graph to illustrate the information given. 			
		Using (v − u)/ t = a to find v			1
		This implies that v = at = 0.2 (1 X 60) = 12 ms^{-1}	1	1	
		The first point at the end of the acceleration would be = ½ *vt = ½ * 12 * 60 = 360m	1	1	
		The additional distance travelled at the speed of 12 ms^{-1} before the 2^{nd} landmark = 1500m − 360m = 1140m		1	1
		Time taken to travel the additional distance at the constant speed = distance/speed = 1140/12 = 95 s		1	1
		Therefore time to get to the 2^{nd} landmark = 60 + 95 = 155 seconds = 2 minutes 35 seconds	1		
TOTAL			**9**	**12**	**9**

Caribbean Examinations Council
Prince Road, Pine Plantation Road,
St Michael BB11091
Tel: (246) 227 1700
Email: cxcezo@cxc.org